C.U.SHAH UNIVERSITY
Summer Examination-2016

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT1   **Branch :** Diploma(All)

Semester : 2   **Date :** 06/05/2016   **Time :**10:30 To 1:30   **Marks :**70

Instructions:
    (1) Use of Programmable calculator & any other electronic instrument is prohibited.
    (2) Instructions written on main answer book are strictly to be obeyed.
    (3) Draw neat diagrams and figures (if necessary) at right places.
    (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**

  a)   The distance between the points $(1, 3)$ and $(0, -4)$ is _____.
    (a) $5\sqrt{2}$ (b) $2\sqrt{5}$ (c) 50 (d) None of these

  b)   Midpoint of $(2,-7)$ and $(8,3)$ is _____.
    (a) $(5,2)$ (b) $(-5,2)$ (c) $(5,-2)$ (d) $(-5,-2)$

  c)   Slope of the line $x+y-8=0$ is _____.
    (a) 1 (b) -1 (c) 8 (d) -8

  d)   Radius of the circle $2x^2 + 2y^2 = 4$ is _____.
    (a) 4 (b) 2 (c) $\sqrt{2}$ (d) None of these

  e)   x – Intercept of line $2x - 6y + 4 = 0$ is _____.
    (a) -2 (b) $\dfrac{1}{3}$ (c) $\dfrac{2}{3}$ (d) None of these

  f)   If $f(x) = x^2 - 1$ then $f(-1) =$ _____.
    (a) 2 (b) 0 (c) -1 (d) None of these

  g) $\lim\limits_{x \to 0} \dfrac{\sin 4x}{\tan 7x} =$ _____
    (a) $\dfrac{4}{7}$ (b) $\dfrac{7}{4}$ (c) 1 (d) None of these

  h) $\dfrac{d\left(\sqrt{x}\right)}{dx} =$ _____
    (a) $2\sqrt{x}$ (b) $\dfrac{1}{\sqrt{x}}$ (c) $\dfrac{-1}{x^2}$ (d) $\dfrac{1}{2\sqrt{x}}$

i) $\dfrac{d(\cos x)}{dx} = $ _____

(a) $-\sec x$ (b) $\sec x$ (c) $-\sin x$ (d) $\sin x$

j) $\dfrac{d(\tan^{-1} x)}{dx} = $ _____

(a) $\dfrac{1}{1+x^2}$ (b) $\dfrac{-1}{1+x^2}$ (c) $\dfrac{1}{\sqrt{1-x^2}}$ (d) $\dfrac{-1}{\sqrt{1-x^2}}$

k) $\int x^n dx = $ _____

(a) $x^{n-1} + c$ (b) $nx^{n-1} + c$ (c) $nx^n + c$ (d) None of these

l) $\int \dfrac{1}{x^2+1} dx = $ _____

(a) $\tan^{-1} x + c$ (b) $\sin^{-1} x + c$ (c) $\cos^{-1} x + c$ (d) $\cot^{-1} x + c$

m) $\int \dfrac{1}{\sqrt{x^2-a^2}} dx = $ _____

(a) $\cot^{-1}\dfrac{x}{a} + c$ (b) $\tan^{-1}\dfrac{x}{a} + c$ (c) $\log\left|x + \sqrt{x^2-a^2}\right| + c$ (d) none of these

n) $\int\limits_{2}^{5} \dfrac{1}{x} dx = $ _____

(a) $\log\dfrac{2}{5}$ (b) $\log\dfrac{5}{2}$ (c) $\log 10$ (d) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Show that the points $(4, 8)$, $(4, 12)$ and $(4 + 2\sqrt{3}, 10)$ are the vertices of an equilateral triangle. **(5)**

b) Prove that $\lim\limits_{x\to\infty} \sqrt{x^2+2x} - \sqrt{x^2-3} = 1$. **(5)**

c) Find $\dfrac{dy}{dx}$ if $y = \dfrac{x^2-1}{x^2+1}$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Find the equation of a circle passing through point $(-7, 1)$ and centre $(-4, -3)$. **(5)**

b) Find $\dfrac{dy}{dx}$ if $y = \log\sqrt{\dfrac{a+x}{a-x}}$ **(5)**

c) Evaluate: $\int \dfrac{\cos(\log x)}{x} dx$ **(4)**

Q-4 **Attempt all questions** **(14)**

a) Find $\dfrac{dy}{dx}$ if $e^x + e^y = e^{x+y}$. **(5)**

b) Evaluate: $\lim\limits_{x \to 3} \dfrac{x^3 - 27}{\sqrt[3]{x} - \sqrt[3]{3}}$ **(5)**

c) Show that (3, 2), (5, 4) and (7, 6) are collinear. **(4)**

Q-5 **Attempt all questions** **(14)**

a) Using definition, find derivative of $y = \sin^2 x$. **(5)**

b) Evaluate: $\int x^2 \log x \, dx$ **(5)**

c) Evaluate: $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{1 - \sin x}{\cos^2 x}$ **(4)**

Q-6 **Attempt all questions** **(14)**

a) Find co ordinates of the points of trisection of the line segment joining points A(4, 4) and B(–2, 1). **(5)**

b) If $f'(x) = 4x^2 + 6x - 3$ and $f(1) = 2$ then find function $f(x)$. **(5)**

c) The equation of motion of a particle is $S = 2t^3 + 3t^2 - 12t + 5$. **(4)**
(i) Find velocity at t = 0. (ii) Find acceleration at t = 1.

Q-7 **Attempt all questions** **(14)**

a) If $y = e^x \sin x$ then prove that $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$. **(5)**

b) Prove that $\int\limits_{0}^{\frac{\pi}{2}} \dfrac{\tan x}{\tan x + \cot x} \, dx = \dfrac{\pi}{4}$. **(5)**

c) If radius of a circle $2x^2 + 2y^2 - 4x - 8y + k = 0$ is 4, find value of k. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Find the area of the region bounded between curve $y = x^2$ and straight-line $x = 2$. **(5)**

b) Find angle between straight lines $\sqrt{3}x - y + 1 = 0$ and $x - \sqrt{3}y + 2 = 0$. **(5)**

c) If $f(x) = \log_2 x$, $g(x) = x^4$ then prove that $f(g(2)) = 4$. **(4)**